Hex Player—A Virtual Musical Controller

Andrew J. Milne	Anna Xambó	Robin Laney
Computing Department	Computing Department	Computing Department
The Open University	The Open University	The Open University
Milton Keynes, UK	Milton Keynes, UK	Milton Keynes, UK
andymilne@tonalcentre.org	a.xambo@open.ac.uk	r.c.laney@open.ac.uk
David B. Sharp	**Anthony Prechtl**	**Simon Holland**
Department of DDEM	Independent Researcher	Computing Department
The Open University	California, USA	The Open University
Milton Keynes, UK	aprechtl@gmail.com	Milton Keynes, UK
d.sharp@open.ac.uk		s.holland@open.ac.uk

ABSTRACT

In this paper, we describe a playable musical interface for tablets and multi-touch tables. The interface is a generalized keyboard, inspired by the Thummer, and consists of an array of virtual buttons. On a generalized keyboard, any given interval always has the same shape (and therefore fingering); furthermore, the fingering is consistent over a broad range of tunings. Compared to a physical generalized keyboard, a virtual version has some advantages—notably, that the spatial location of the buttons can be transformed by shears and rotations, and their colouring can be changed to reflect their musical function in different scales.

We exploit these flexibilities to facilitate the playing not just of conventional Western scales but also a wide variety of microtonal generalized diatonic scales known as moment of symmetry, or well-formed, scales. A user can choose such a scale, and the buttons are automatically arranged so their spatial height corresponds to their pitch, and buttons an octave apart are always vertically above each other. Furthermore, the most numerous scale steps run along rows, while buttons within the scale are light-coloured, and those outside are dark or removed.

These features can aid beginners; for example, the chosen scale might be the diatonic, in which case the piano's familiar white and black colouring of the seven diatonic and five chromatic notes is used, but only one scale fingering need ever be learned (unlike a piano where every key needs a different fingering). Alternatively, it can assist advanced composers and musicians seeking to explore the universe of unfamiliar microtonal scales.

Keywords

generalized keyboard, isomorphic layout, multi-touch surface, tablet, musical interface design, iPad, microtonality

1. INTRODUCTION

Hex Player is a virtual musical controller. It is played by the fingers and sends standard MIDI messages to control any software or hardware synthesizer. It is designed to make playing music easier without imposing a ceiling on
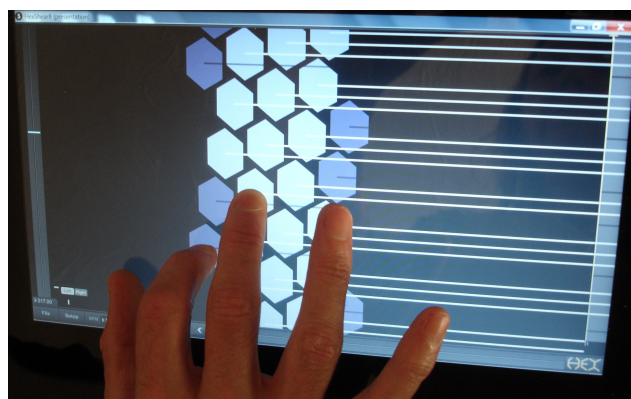
Figure 1: Hex Player on a tablet. The white/grey buttons are generalized diatonic/chromatic notes.

expressiveness and creative possibilities. Hex Player lives on a multi-touch surface such as a tablet (e.g., iPad) or table (e.g., Evoluce or Microsoft Surface), and consists of a lattice (array) of hexagonal buttons (see Figure 1).

Pressing any of these virtual buttons sends a MIDI note event to a software, or external hardware, synthesizer. The blank areas to the left and right of the buttons are a control surface that can be operated by the thumb or pinkie (little finger) of each hand; these can be used to control a number of expressive parameters such as timbre, volume, vibrato, or tuning, while the four fingers (or three fingers and thumb) play notes. This means that an expressive lead part (with pitch bends, and vibrato, etc.) can be played with one hand, and a bass line or chords with the other. Contrast this with a piano-style keyboard, where an expressive lead part typically requires two hands—one to play the notes, the other to operate the pitch-bend/modulation wheel/joystick.

Hex Player's note layouts and use of a thumb-operated controller are based upon the design of Thumtronic Inc.'s prototypical hardware instrument, the Thummer (`http://thummer.com`). The Thummer project is now open source and, in this paper, we describe how we have transferred many of the design features of the hardware Thummer to a software virtual interface, and how we have further extended its capabilities in ways that would not be possible in a hardware device.

Like the Thummer, Hex Player provides a generalized keyboard and uses an isomorphic note layout. Generalized keyboards have their keys, or buttons, arranged in a regular (typically two-dimensional) lattice. Such keyboards date back at least as far as the nineteenth century; many exam-

ples of which are given in [6]. On a generalized keyboard, it is possible to arrange pitches isomorphically, which ensures that the same interval, scale, or chord, always has the same geometrical shape regardless of transposition [2, 7]. Furthermore, intervals, scales, and chords (when categorized according to reasonable criteria) have the same geometrical shape over a wide range of tunings, thus enabling tuning to be altered without affecting note layout [8].

In this paper, we describe four novel extensions to the current theory of generalized keyboards and isomorphic note layouts: firstly, we introduce a class of *adjacent seconds* note layouts, which generalize many of the useful properties of the Wicki accordion button layout (the layout used by the Thummer) over a wide variety of microtonal scales; secondly, we describe how shears and rotations of the layouts ensure the pitch height of each button is shown by its spatial height, and that buttons an octave apart are vertically aligned; thirdly, we show how the amount of shear applied to the layout (and hence its tuning) can be dynamically controlled while playing; fourthly, we show how alterable button colouration can be used to indicate generalized diatonic and chromatic scales (well-formed or MOS scales—formally defined in Section 2.1.4).

In the next section, we provide some of the music theoretical and mathematical underpinnings required to understand these innovations. After that, we discuss some of the potential benefits of an interface like Hex Player.

2. GENERALIZED KEYBOARDS AND ISOMORPHIC NOTE LAYOUTS

Generalized keyboards with isomorphic layouts have a number of properties that may facilitate the comprehension and playing of music.

2.1 Basic Definitions
Before proceeding to a full description of the interface and its properties, some formal definitions may be helpful.

2.1.1 *Generalized Keyboard*
As pictured in Figure 2, a *generalized* keyboard or *button-lattice* consists of a regular array of (real or virtual) buttons [7], each of which plays a musical tone. The buttons could be arranged in one-, two-, or three-dimensional space, but we will restrict this discussion to two-dimensional lattices (because these can be implemented on a surface).

2.1.2 *Two-dimensional Tuning System*
A *two-dimensional tuning system* is one that is generated from two intervals—a *period* (typically the octave), and a *generator* (typically a perfect fifth). For example, the pentatonic scale can be generated by stacking four perfect fifths (e.g., C–G–D–A–E) and reducing them by octaves so all tones lie within one octave (e.g., in pitch order, C, D, E, G, A); the diatonic scale can be generated by stacking six perfect fifths (F–C–G–D–A–E–B) and reducing them by octaves (e.g., in pitch order, C, D, E, F, G, A, B); the chromatic scale by stacking eleven such fifths (e.g., E♭–B♭–F–C–G–D–A–E–B–F♯–C♯–G♯) and reducing them (e.g., in pitch order, C, C♯, D, E♭, E, F, F♯, G, G♯, A, B♭, B). The period and generator can, however, take any size (not just octave and fifth), and different sizes can generate unfamiliar scales that share a number of properties with the familiar pentatonic, diatonic, and chromatic.

2.1.3 *Isomorphic Layout*
An *isomorphic layout* is one in which the period and generator of the tuning system are mapped to a spatial basis of
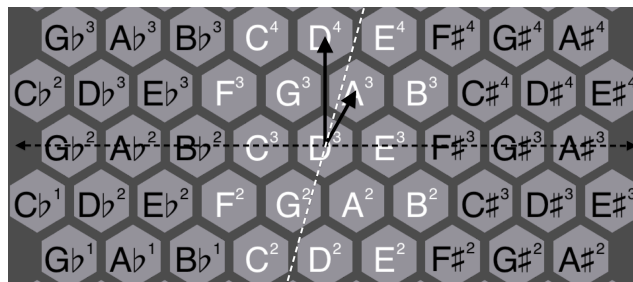

Figure 2: An isomorphic note layout (the Wicki layout) on a virtual generalized keyboard.

the button-lattice [8]. There are several possible bases, and hence several possible mappings (and each different mapping constitutes a different note layout), but one of the most historically successful (at least in the case of the chromatic scale) can be found in the Wicki accordion button layout [13]. The key to understanding the Wicki mapping is to note how the period and generator (here octave and fifth) are mapped spatially (see Figure 2—the arrows show the spatial vectors corresponding to the period and generator).

2.1.4 *Moment of Symmetry/Well-formed Scales*
A *moment of symmetry* (MOS) [14] or *well-formed* [4] scale is a generated scale containing exactly two step sizes that are distributed with maximal evenness. A *generated scale* is produced by repeatedly adding a generator interval (typically a perfect fifth) and then reducing all such intervals by repeatedly subtracting a period interval (typically the octave) so all intervals are smaller than the period.

The number of times the generator can be stacked so as to produce just two step sizes depends on the ratio of the generator and period. For example, if the generator is 702 cents and the period is 1200 cents (a generator/period ratio of 0.585), then well-formed scales are available with 3 tones (e.g., C, D, G), 5 tones (e.g., C, D, E, G, A), 7 tones (e.g., C, D, E, F♯, G, A, B), 12 tones (e.g., C, C♯, D, D♯, E, E♯, F♯, G, G♯, A, A♯, B), 17 tones, and so forth [4]. A different generator/period ratio requires different numbers of tones to produce a well-formed scale—when the generator is 316 cents (a just intonation "minor third") and the period is 1200 cents, the following numbers of tones are well-formed: 3, 4, 7, 11, 15, 19, and so forth. With no loss of generality, whenever the size of the period is not explicitly mentioned, it is assumed to be 1200 cents.

MOS scales have a number of properties that are thought to give them æsthetic value. There is not space to discuss these properties in depth but, briefly: every scale span—generic interval size—comes in exactly two specific interval sizes (Myhill's property [5]); the two scale step sizes are evenly distributed throughout the period; within the period, every scale degree has a unique pattern of intervals surrounding it [1]—this may be necessary for any scale to support tonal functionality; when transposed by the generator, the resulting scale shares all but one tone, facilitating modulation [1]; collectively, these features suggest a good compromise between the excessive simplicity of equal step scales and the complexity of completely irregular scales [3].

An MOS, therefore, provides an effective way to choose a scale (a set of notes) from any abstract 2-D tuning.

2.2 General Properties of Isomorphic Layouts
An isomorphic layout has a number of musically useful properties discussed in the subsections below.

2.2.1 Transpositional Invariance

Any given interval, chord, or scale, has the same geometrical shape (and hence fingering) regardless of its location (transposition) on the keyboard. So, a musician need learn the fingering for a major scale, or harmonic minor scale, or third inversion of a dominant seventh chord, just once, and then apply that same shape to any key or root note. Compare this to the piano, where every different major scale requires a different pattern of notes to be memorized [8].

2.2.2 Tuning Invariance

The fingering of a wide range of scales can remain invariant over a wide range of tunings. For example, the fingering of diatonic/chromatic music will stay essentially invariant when the generator has any size between 685.714 and 720 cents. This continuum of tunings includes many notable tunings, such as 19-tone equal temperament (19-TET), various meantone tunings, 12-TET, Pythagorean, 17-TET and 22-TET, so consistent fingering may facilitate the exploration of alternative tunings [8]. There are many different tuning continua, each of which smoothly connects a wide variety of notable tunings [9].

2.2.3 Pitch Axis

Any isomorphic layout has a pitch axis, and the position of any button centre in relation to this axis indicates its pitch. The angle of this axis depends on the note layout (spatial mapping of the period and generator) used and the tuning ratio of the generator and period [9]. For example, in Figure 2, the pitch axis for a generator of 700 cents (which corresponds to a 12-TET tuning) is identified by the dashed white line—note how the distance, measured along this line, between D3 and E3 is two thirds of the distance between D3 and F3 (draw three lines between the pitch axis and D3, E3, and F3, such that all three lines are orthogonal to the pitch axis; the distance between the D3 and E3 lines is two thirds the distance between the D3 and F3 lines).

2.2.4 Generator-Span Axis

The generator-span axis is a novel concept introduced in this paper. Any isomorphic layout also has a generator-span axis, such that the distance between any two button centres, as measured on this axis, indicates the number of generators between them. For example, in Figure 2 the generator-span axis is shown with a black dashed line—the distance, measured on this axis, between D3 and A3 is half the distance of D3 and E3 (there is one fifth between the former, and two fifths between the latter).

Assuming the period and generator are linearly independent, any two notes a period (octave) apart have zero generator distance, hence the generator-span axis is orthogonal to the spatial mapping of the period.

Pitch distance and generator distance are two important musical metrics. The importance of the former is obvious; one importance of the latter is that any given MOS scale always forms a strip running parallel with the octaves.

3. HEX PLAYER

In Hex Player, the pitch and generator-span axes are oriented, and a specific isomorphic layout is selected, so as to maximize certain useful criteria described below.

3.1 Orthogonal Axes

Due to their importance, we have endeavoured to make the pitch and generator-span axes easy to discern, visually and haptically. To do this, we use a novel approach: applying shear and rotation transforms of the lattice to make the pitch axis vertical and the generator-span axis horizontal, regardless of the isomorphic layout or tuning being used. The vertical pitch axis means it is easy to know, in advance of playing, the pitch of any button and that, as the hand plays an ascending scale, it moves gradually away from the body. Furthermore, the horizontal generator-span axis ensures that notes an octave apart are vertically aligned.

3.2 Adjacent Seconds Layouts

Adjacent seconds layouts are a novel concept, and generalize, for any possible MOS scale, some of the useful features of the Wicki layout: a notable property of the Wicki layout, when used for the pentatonic and diatonic scales, is that the most numerous scale step (the whole-tone) runs along each row (in Figure 2, observe the scale runs C-D-E and F-G-A-B), while the least numerous scale step ("minor third" in the pentatonic, and semitone in the diatonic) is reached by a "carriage return" skip up to the next row (in Figure 2, observe the steps E-F and B-C).

For the Wicki layout, this neat property breaks down for most other MOS scales. For example, Figure 3a shows the Wicki layout of the MOS scale with 4 large and 7 small steps (the generator is 320 cents)—the most numerous seconds now skip buttons, the scale's pattern is more difficult to make out, and is not spatially compact. However, by choosing an appropriate isomorphic layout, it is possible for any MOS scale to have the adjacent seconds property. Indeed, for the 120 different possible MOS scales with nineteen or fewer tones, only 13 different layouts are required.

Figure 3b shows how an adjacent seconds layout for the 4 large 7 small MOS scale gives a far more compact and easy to understand layout than the Wicki. In Hex Player, a user can first choose an MOS scale (by inputting the number of large and small steps in the scale), and then click on "Optimize Layout" to switch to an adjacent seconds layout.

3.3 Dynamic Tuning

The horizontal generator-span axis ensures all MOS scales form a vertical strip of buttons. The space on either side can, therefore, be used as a control surface accessible to the thumb and pinkie; either of these fingers can change the shear of the lattice, and send a correlated MIDI CC value, while the remaining fingers are playing. The CC message can ensure the synth's tuning matches that implied by the lattice's shear.[1] This opens up the possibility of players dynamically mimicking the expressive intonations used by advanced string and aerophone players. For example, the thumb can be used to move smoothly between meantone tunings that are ideal for sustained chords, and Pythagorean tunings suitable for expressive melodies [11].

Outside of these familiar Western tunings, the performer can move dynamically, and smoothly, to non-Western tunings such as 5-TET (as used in Indonesian slendro [12]) or 7-TET (as used in traditional Thai music [10]).

These large thumb-/pinkie-operated control surface areas can also be mapped to any parameter, so may also control other pitch or timbral features (e.g., vibrato or brightness).

3.4 Generalized Chromatic Embeddings

Any MOS scale with m large steps and n small can be embedded in an MOS scale with $2m+n$ steps. For example, the pentatonic scale can be embedded within the diatonic scale, which can be embedded within the chromatic, and so on [4]. This provides a neat method to generalize the

---

[1]Dynamic tuning changes such as these require the use of a Dynamic Tonality synthesizer such as TransFormSynth, The Viking, and 2032, which can be downloaded from `http://www.dynamictonality.com`.
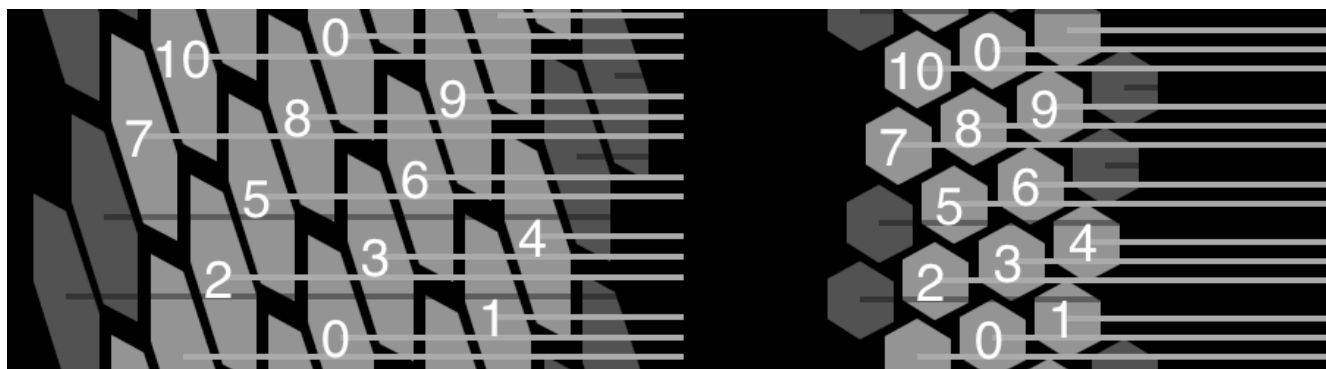
Figure 3: The scale degrees of the 4 large, 7 small steps MOS scale (the generator is 320 cents) in a) the Wicki layout, and b) an adjacent seconds layout. The horizontal lines show the pitch height of every button.

"chromatic" embedding of any MOS scale. In Hex Player, when an MOS scale is chosen (by entering the number of large and small steps), it is automatically displayed as light-coloured buttons (in a central vertical band), surrounded by dark-coloured "chromatic" notes.

This allows for scales of increasing complexity to be presented to a beginner in a consistent fashion. For example, children are frequently taught the pentatonic scale (e.g., C, D, E, G, A) as a first step in their musical education (e.g., the Orff and Kodály methods). In Hex Player, the pentatonic scale can be shown as a light coloured vertical strip of buttons, while the more challenging diatonic notes (in this case, F and B) are dark-coloured buttons on either side of this strip. When the student is ready, the diatonic scale can be shown as vertical strip of light-coloured buttons, with the more challenging chromatic tones shown in a dark colour either side. All of these representations are consistent—the same spatial shape always plays the same interval—but, as the scales become more complex, the strip just gets wider.

4. CONCLUSIONS

This paper presents a novel virtual musical interface designed to facilitate the playing of both familiar and unfamiliar musics by musicians at all levels: from beginners to advanced microtonalists. We do this by combining many of the well-established advantages of generalized keyboards with some novel extensions.

In summary, the interface allows a user to select any MOS scale, and its tuning, such that: a) all intervals have the same shape regardless of transposition; b) all intervals (categorized by reasonable criteria) have the same shape over a wide range of tunings; c) the layout ensures the pitch axis and generator-span axis are vertical and horizontal, respectively, and so are visually and haptically salient and distinct; d) the most numerous scale steps run along rows, the less numerous are "carriage returns" up to the next row; e) the notes in the scale and the "chromatic" scale, within which it is embedded, are displayed as light- and dark-coloured buttons lying in a vertical strip at the centre of the display; f) the space either side of the note strip can be used as a control surface, enabling the thumb or pinkie to dynamically alter the tuning (and correspondingly change the shear of the lattice so as to reflect the resulting pitches of the buttons) while the remaining fingers play.

The dynamically changing shears, rotations, and button colourings are difficult to implement in a hardware interface, hence the usefulness of the virtual realisations made possible by multi-touch surfaces. A drawback of current surfaces, however, is the lack of tactile feedback and velocity and pressure sensitivity. It is unknown, at this stage, to what extent this impacts upon their utility; we intend to explore these issues in future research.

5. ACKNOWLEDGMENTS

6. REFERENCES

[1] G. J. Balzano. The pitch set as a level of description for studying musical perception. In M. Clynes, editor, *Music, Mind, and Brain.* Plenum Press, New York, 1982.

[2] R. H. M. Bosanquet. *Elementary Treatise on Musical Intervals and Temperament.* Macmillan, London, 1877.

[3] N. Carey. Coherence and sameness in well-formed and pairwise well-formed scales. *Journal of Mathematics and Music*, 1(2):79–98, 2007.

[4] N. Carey and D. Clampitt. Aspects of well-formed scales. *Music Theory Spectrum*, 11(2):187–206, 1989.

[5] J. Clough, N. Engebretsen, and J. Kochavi. Scales, sets, and interval cycles: A taxonomy. *Music Theory Spectrum*, 21(1):74–104, 1999.

[6] H. L. F. Helmholtz. *On the Sensations of Tone as a Physiological Basis for the Theory of Music.* Dover, New York, 1877.

[7] D. Keislar. History and principles of microtonal keyboards. *Computer Music Journal*, 11(1):18–28, 1987.

[8] A. J. Milne, W. A. Sethares, and J. Plamondon. Isomorphic controllers and Dynamic Tuning: Invariant fingering over a tuning continuum. *Computer Music Journal*, 31(4):15–32, 2007.

[9] A. J. Milne, W. A. Sethares, and J. Plamondon. Tuning continua and keyboard layouts. *Journal of Mathematics and Music*, 2(1):1–19, 2008.

[10] D. Morton. The music of Thailand. In E. May, editor, *Music of Many Cultures*, pages 63–82. University of California Press, Berkeley, CA, 1980.

[11] J. Sundberg, A. Friberg, and L. Frydén. Rules for automated performance of ensemble music. *Contemporary Music Review*, 3(1):89–109, 1989.

[12] W. Surjodiningrat, P. J. Sudarjana, and A. Susanto. *Tone Measurements of Outstanding Javanese Gamelan in Yogyakarta and Surakarta.* Gadjah Mada University Press, Yogyakarta, Indonesia, 1993.

[13] K. Wicki. Tastatur für musikinstrumente, Oct. 1896.

[14] E. Wilson. Letter to Chalmers pertaining to moments-of-symmetry/Tanabe cycle, 1975.